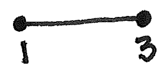
Math 3305, Chapter 1, Section 3 Script

Digging right in to all this vocabulary – much will be familiar! Have your book handy for this video! You'll need for a popper answer soon!

A word about line segments and distance.

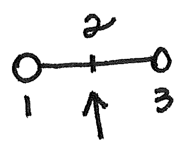
Notation: a line segment that includes its endpoints

The segment from 1 to 3 with the endpoints: $1 \le x \le 3$



and one that doesn't

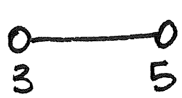
The segment from 1 to 3 without the endpoints: $1 < x < 3$



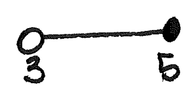
2 is an interior point of the segment and it's midpoint.

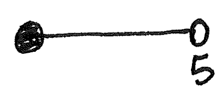
Popper 1.3, Question One

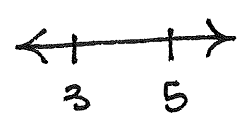
What does the segment $3 < x \le 5$ look like in line segment notation?

A. 

B. 

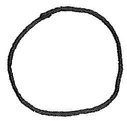
C. 

D. 

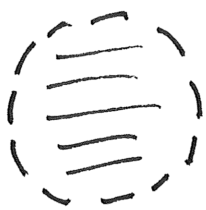
E. 

Circles, used to be in a Postulate but now it's a definition:

The set of all points equidistant from a given point, its center.



Interior points are different from the circle. Convexity!



like Klein Disc

or HG Poincaré Disc

Be sure to review English and Metric units on page 11. Units analysis:

$\pi = 180°$ $\quad \frac{\pi}{180°} = 1 \quad 1 = \frac{180}{\pi} \quad 30° \cdot 1 =$

$30° \cdot \frac{\pi}{180°} = \frac{\pi}{6}$ "rads"

1 yard = 3 feet $\quad \frac{1\text{ yd}}{3\text{ ft}} = 1 \quad \frac{3\text{ feet}}{1\text{ yd}} = 1$

1 mile = 5280 feet = 1.6093 km $\quad \frac{1\text{ mile}}{5280\text{ feet}} = 1 \quad \frac{1.6093\text{ km}}{5280\text{ feet}} = 1$

Popper 1.3, Question Two

6 inches = _______yards

A. $6\text{ in} \cdot \frac{36\text{ in}}{1\text{ yd}}$

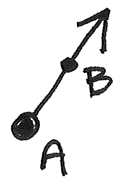
B. $6\text{ in} \cdot \frac{1\text{ yd}}{36\text{ in}}$

C. $\frac{1}{6\text{ in}} \cdot \frac{1\text{ yd}}{36\text{ in}}$

D. can't be done

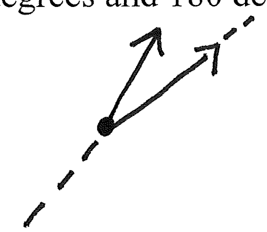
Now rays and angles:

Ray: a closed half-line with endpoint A and interior point B

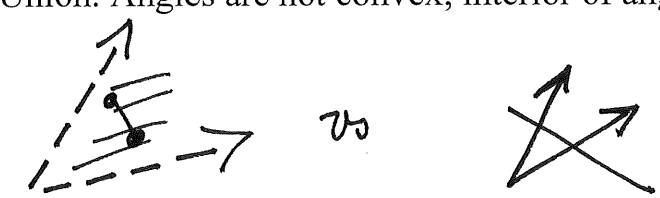


Angle: a pair of rays sharing a common endpoint. A union of the points of the rays.

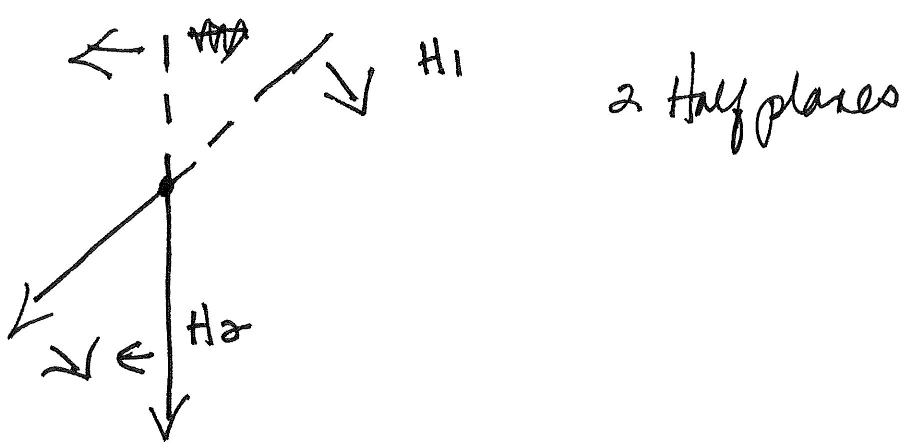
SMSG Axiom 11: To every angle there corresponds a real number between zero degrees and 180 degrees.



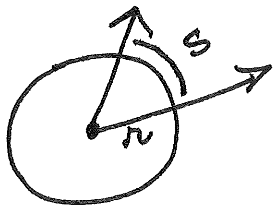
Definition Union. Angles are not convex; interior of angle, though



Interior of an angle, an Intersection. Note no straight angles in our geometry! And no angles measuring zero degrees either. Convexity.

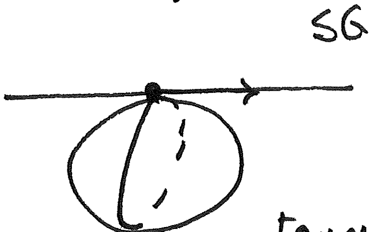


Combining circles and angles, an angle with its vertex at the center cuts off an arc length. The measure of the angle in radian measure is arclength divided by radius length.

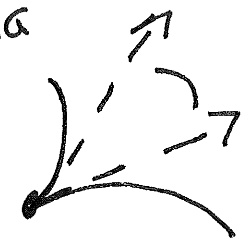


$\frac{\text{arclength}}{r}$ ($\frac{s}{r}$) = m central ∠ in rads

Measuring angles in HG and SG…not for memorizing! Just note that you can do it and the way to do it is different.



tangent plane



tangent lines

just note there's an angle measure angle axiom in each

Let's look at radian measure again. Angle theta

30 degrees $\quad 30^\circ \cdot 1 = 30^\circ \frac{\pi}{180^\circ} = \frac{\pi}{6}$ "rad"

"1"

Pi/4 rads $\quad \frac{\pi}{4} \cdot \frac{180^\circ}{\pi} = 45^\circ$

OYO:

Check out the notation box in the center of page 13! Right now the answer to Popper Question Three is there!

Given the following points, find the equation of the line in standard form. Check your work.

(2, 4) and (4, 7) typical question just the points set $C=1$

$$\left.\begin{array}{l} 2A+4B+1=0 \\ 4A+7B+1=0 \end{array}\right\} \text{step 1}$$

$$\left[\begin{array}{ccc|c} 2 & 4 & 1 & 0 \\ 4 & 7 & 1 & 0 \end{array}\right]$$ zero it out

$-2R1+R2=NR2$

$$\begin{array}{cccc} -4 & -8 & -2 & 0 \\ 4 & 7 & 1 & 0 \\ \hline 0 & -1 & -1 & 0 \end{array}$$

$$\left[\begin{array}{ccc|c} 2 & 4 & 1 & 0 \\ 0 & -1 & -1 & 0 \end{array}\right]$$

$4R2+R1=NR1$

$$\left[\begin{array}{ccc|c} 0 & -4 & -4 & 0 \\ 2 & 4 & 1 & 0 \end{array}\right]$$

$2 \quad 0 \quad -3 \quad 0$

$$\left[\begin{array}{ccc|c} 2 & 0 & -3 & 0 \\ 0 & -1 & -1 & 0 \end{array}\right]$$

$2A-3=0 \quad A=3/2$

$-1B-1=0 \quad B=-1$

claim

$3/2\,x-y+1=0 \quad 3x-2y+2=0$

check $(2,4)$

$3(2)-2(4)+2=0$

$6-8+2$ ✓

$(4,7)$

$3(4)-2(7)+2=0$

$12-14+2=0$ ✓

Popper 1.3, Question Four

What's the first step to do below?

A. Write out the equation. It's obvious, isn't it?

B. Identify A and B

C. Fill in two standard equations with what we know: x and y

Popper 1.3, Question Five

Which of the following is in standard form?

A. $y = mx + b$

B. $(y - y1) = m(x - x\ 1)$

C. $ax + by + c = 0$

Popper 1.3, Question Three

AB means

A. Ray AB

B. Line AB

C. The distance from A to B

D. Line segment AB

Now some work with lines. Two points determine a line…Axiom 1. Let's look at how to get the equation of a line. Let's look at the FORMS of the equations

Standard $Ax + By + C = 0$

Slope intercept $y = mx + b$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1} \quad (0, b)$

Two point $y - y_1 = m(x - x_1)$

Now let's look slowly, carefully at a some of examples of how to get the equation a new way. This foreshadows some work with matrices and may not be familiar to you.

$2x - y + 3 = 0$ STD FORM

Let's write down the steps. Using y = 2x + 3 so we know our goal. JUST THIS ONCE

Given points (1, 5) and (2, 7). Find the equation of the line. Check your work.

Identify what you know. The x's and y's. Put them in standard form. Note you don't know the A's and B's!

(1,5) $1A + 5B + C = 0$ $\underline{C = 1}$

(2,7) $2A + 7B + C = 0$

Abstract the standard form to a matrix suppressing lots!

$$\left[\begin{array}{ccc|c} 1 & 5 & 1 & 0 \\ 2 & 7 & 1 & 0 \end{array}\right]$$

Use row operations to get A and B. Put the answer in standard format.

want ↑ that 2 to be a zero

$-2R1 + R2 = NR2$

$$\begin{array}{cccc} -2 & -10 & -2 & 0 \\ 2 & 7 & 1 & 0 \\ \hline \end{array}$$

NR2 $0 \quad -3 \quad -1 \quad 0$

zero that

$$\left[\begin{array}{ccc|c} 1 & 5 & 1 & 0 \\ 0 & -3 & -1 & 0 \end{array}\right]$$

Goal: $\left[\begin{array}{ccc|c} 1 & 0 & \checkmark & 0 \\ 0 & 1 & \checkmark & 0 \end{array}\right]$

$\frac{5}{3} R2 + R1 = NR1$

$$\begin{array}{cccc} 1 & 5 & 1 & 0 \\ 0 & -5 & -5/3 & 0 \\ \hline \end{array}$$

NR1 $1 \quad 0 \quad -2/3 \quad 0$

$$\begin{bmatrix} 1 & 0 & -2/3 & 0 \\ 0 & -3 & -1 & 0 \end{bmatrix}$$

$A - 2/3 = 0$

$-3B - 1 = 0$

$A = 2/3$

$B = -1/3$

ANS

$2/3x - 1/3y + 1 = 0$ mult by 3 BS

$2x - y + 3 = 0$

Check your work.

(1,5) $2 - 5 + 3 \checkmark$

(2,7) $4 - 7 + 3 \checkmark$

Now for another problem. Given these two points: ~~(2, 4)~~ (1,4) and ~~(4, 7)~~ (2,3), write the equation of the line.

$(1,4) \quad A + 4B + C = 0$

$(2,3) \quad 2A + 3B + C = 0$

$\rightarrow$

2ND

$$\left[\begin{array}{ccc|c} 1 & 4 & 1 & 0 \\ 2 & 3 & 1 & 0 \end{array}\right]$$

1ST

$-2R1 + R2 = NR2$

$$\begin{array}{lcccc} & -2 & -8 & -2 & 0 \\ & 2 & 3 & 1 & 0 \\ \hline NR2 & 0 & -5 & -1 & 0 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 4 & 1 & 0 \\ 0 & -5 & -1 & 0 \end{array}\right]$$

$\frac{4}{5}R2 + R1 = NR1$

$$\begin{array}{lcccc} & 1 & 4 & 1 & 0 \\ & 0 & -4 & -4/5 & 0 \\ \hline NR1 & 1 & 0 & 1/5 & 0 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1/5 & 0 \\ 0 & -5 & -1 & 0 \end{array}\right]$$

$A + 1/5 = 0 \qquad A = -1/5$

$-5B - 1 = 0 \qquad B = -1/5$

$-\frac{1}{5}x - \frac{1}{5}y + 1 = 0 \qquad$ M.BS. by -5

$x + y - 5 = 0$

$\downarrow$

ck

$(1,4) \qquad 1 + 4 - 5$ ✓

$(2,3) \qquad 2 + 3 - 5$ ✓

And another one with (1, -8) and (2, -13)

$1A - 8B + 1 = 0$
$2A - 13B + 1 = 0$

$$\left[\begin{array}{ccc|c} 1 & -8 & 1 & 0 \\ 2 & -13 & 1 & 0 \end{array}\right]$$

$-2R1 + R2 = NR2$

-2	16	-2	0	
2	-13	1	0	
0	3	-1	0	NR2

$$\left[\begin{array}{ccc|c} 1 & -8 & 1 & 0 \\ 0 & 3 & -1 & 0 \end{array}\right]$$

$8/3\, R2 + R1 = NR1$

0	$+8$	$-8/3$	0	
1	-8	$3/3$	0	
1	0	$-5/3$	0	NR1

$$\left[\begin{array}{ccc|c} 1 & 0 & -5/3 & 0 \\ 0 & 3 & -1 & 0 \end{array}\right]$$

$A - 5/3 = 0$

$3B - 1 = 0$

$A = 5/3 \quad B = 1/3$

$5/3\, x + 1/3\, y + 1 = 0$

mult BS by 3

$5x + y + 3 = 0$

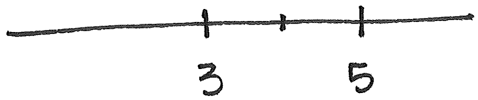
ck $(1, -8)$ $\qquad$ $(2, -13)$

$5(1) - 8 + 3$ $\qquad$ $5(2) - 13 + 3 = 0$

Moving along now, let's look at the SMSG A3, part C distance formula.

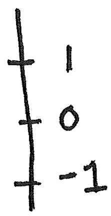
The distance from a Point A with coordinate a to a Point B with coordinate b is $|a-b|$.

Horizontally



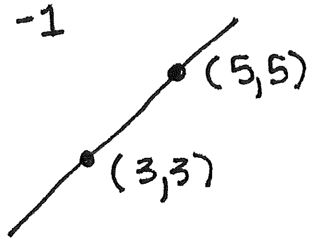
$|5-3| = 2$

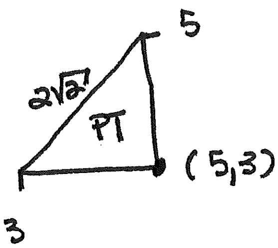
Vertically



$|1 - -1| = 2$

Oblique





Now the Euclidean Distance Formula with (x, y) coordinates

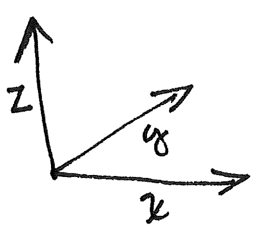
$$\sqrt{(3-5)^2 + (3-5)^2} = \sqrt{4+4} = \sqrt{8} \quad 2\sqrt{2} \checkmark$$

Can we reverse the subscripted variables? yes $(5-3)^2 = 2 = (3-5)^2 = (-2)^2$

Now what about 3D? Let's look at that one!

(x, y, z)

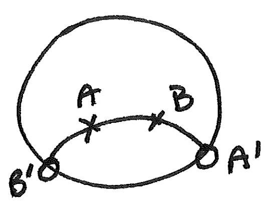
$$\sqrt{(x_1-x_2)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$



And just a bit about the distance formula in Spherical and Hyperbolic Geometries. Each has a distance formula and an axiom allowing that. Let's just look at them for a minute. It's important that you've seen them. Each one of these has an axiom enabling measuring distance, too.

HG

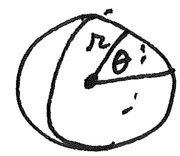
$$\left| \ln \left(\frac{BA' \cdot AB'}{AA' \cdot BB'} \right) \right|$$



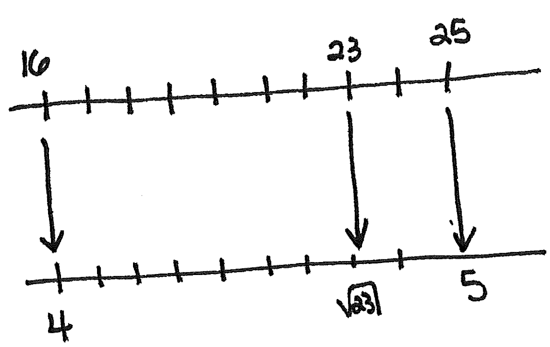
ideal points on the circle

SG

$r\theta$ using the center of the sphere, which isn't a point in the space!



Changing the subject somewhat! Back to Euclidean Geometry and just a bit about square roots. You know you get square roots a whole lot when using that Euclidean distance formula. How big is that number approximately? Let's look at a couple of irrational square roots and find out where they are approximately. What is a nearby rational number to $\sqrt{23}$? How big is this number really?



9 steps 16 → 25

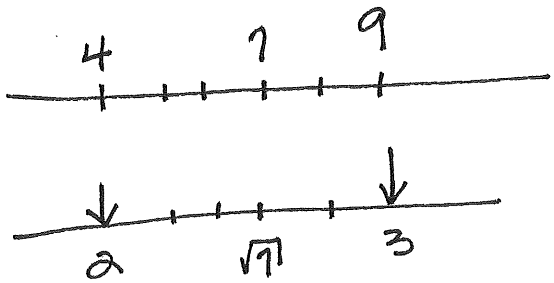
7 steps 4 → $\sqrt{23}$

$$\sqrt{23} \approx 4\tfrac{7}{9}$$

$\approx$ $4.\overline{777}$

4.7958

And the square root of 7?



5 steps

3 step

$2\frac{3}{5} \approx 2.6$

$\sqrt{7} \approx 2.6457$

Now the steps:

Make two horizontal number lines stacked: one a number line including bracketing perfect squares, the other a square root number line.

The smaller square root from the square root line is the whole number.

Count the steps from one perfect square to the next. This is the denominator.

Count the steps from the smaller perfect square to the one in question. This is the numerator.

Make a mixed number. Voila!

Popper 1.3, Question Six

What is the whole number part of the a nearby rational number to $\sqrt{15}$?

A. 3

B. 4

1.3 Essay One

"Number Sense" is a skill that we try to impart to middle school children. Look up "number sense" on the internet and in the middle school TEKS. Write a couple of paragraphs about how the process of finding nearby rational numbers to irrational numbers like the square root of 17 is part of getting skilled at "number sense".

Ok now, wrapping up 1.3:

You have a brief essay to turn in under assignments in CourseWare.

You have a 6 question Popper 1.3 to answer under the EMCF tab.

And:

Homework 1.3, Turn in with all of homework Chapter 1 under the assignments tab.

#6 from this section of the textbook.

1.3 Ms. Leigh One

Find the equation of the line through (2, 5) and (4, 3) the new way and check your work.

1.3 Ms. Leigh Two

Find the rational mixed number that is very near $\sqrt{39}$.